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The Application of Fourier Transform in Determining the Velocity of Two Dimensional (2D) Interfering Waves.

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Abstract: In this paper we considered the propagation of a carrier wave in a free 2D cylindrical coordinate space. A carrier wave as the name implies is the resultant interference of a 'parasitic wave' on a 'host wave'. It is assumed that the initial characteristics of the 'host wave' are known while that of the 'parasitic wave' is not known before the superposition. The attenuation mechanism of the total phase angle, the characteristic angular velocity and the radial velocity of the carrier wave produced by the two interfering waves is effectively studied by using Fourier transform technique. This study provides a method for determining the characteristics of a 'parasitic wave' in the carrier wave whose initial characteristics were not known. It is shown that when the basic features of a carrier wave is undergoing attenuation under any circumstance, they do not consistently come to rest; rather they show some resistance during the decay process, before it is finally brought to rest. The irregular complex behaviour exhibited by the maximum displacement of the carrier wave during the damping process, is due to the resistance pose by the components of the 'host wave' in an attempt to annul the destructive effects of the interfering 'parasitic wave'. Also the inconsistent decay behaviour of the carrier wave is caused by constructive interference (high attraction) or destructive interference (high repulsion) between the 'host wave' and the 'parasitic wave'.

Keywords: Carrier wave, Characteristic angular velocity, Constituted Carrier Wave (C.C.W), Fourier analysis, Host wave and parasitic wave

1.0 Introduction

In physics, a wave is disturbance or oscillation that travels through matter and space, accompanied by a transfer of energy. Wave motion transfers energy from one point to another, often with no permanent displacement of the particles of the medium, that is, with little or no associated mass transfer. They consist, instead, of oscillations or vibrations around almost fixed locations. Waves are described by a wave equation which sets out how the disturbance proceeds over time. The mathematical form of this equation varies depending on the type of the wave [1].

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In physics and systems theory, the superposition principle, also known as superposition property, states that, for all linear systems, the net response at a given place and time caused by two or more stimuli is the sum of the responses which would have been caused by each stimulus individually. The homogeneity and additivity properties together are called the superposition principle. That is, for additivity we have $F(x_1 + x_2 + \ldots) = F(x_1) + F(x_2) + \ldots$, while for homogeneity $F(a\,x) = a\,F(x)$ where a is some scalar [2], [3].

This principle has many applications in physics and engineering because many physical systems can be modelled as linear systems. For example a beam can be modelled as a linear system where the input stimulus is the load on the beam and the output response is the deflection of the beam. The importance of linear systems is that they are easier to analyse mathematically; there is a large body of mathematical techniques, frequency domain linear transform methods such as Fourier, Laplace transforms, and linear operator theory that are applicable. Because physical systems are generally only approximately linear, the superposition principle is only an approximation of the true physical behaviour [4], [5].

The superposition principle applies to any linear system, including algebraic equations, linear differential equations and systems of equations of those forms. The stimuli and response could be numbers, functions, vectors, vector fields, time-varying signals, or any other object which satisfies certain axioms. Note that when vectors or vector fields are involved, a superposition is interpreted as a vector sum. For example, in Fourier analysis, the stimulus is written as the superposition of infinitely many sinusoids [6].

Due to the superposition principle, each of these sinusoids can be analyzed separately, and its individual response can be computed. The response is itself a sinusoid, with the same frequency as the stimulus, but generally a different amplitude and phase. According to the superposition principle, the response to the original stimulus is the sum (or integral) of all the individual sinusoidal responses [7], [8].

The phenomenon of interference between waves is based on the idea of superposition of waves. When two or more waves traverse the same space, the net amplitude at each point is the sum of the amplitudes of the individual waves. In some cases, the summed variation has smaller amplitude than the component variations; this is called destructive interference. In other cases, the summed variation will have bigger amplitude than any of the components individually; this is called constructive interference [3], [9].

Some waves in nature behave parasitically when they interfere with another one. Such waves as the name implies has the ability of transforming the initial characteristics and behaviour of the interfered wave to its own form and quality after a period of time. Under this circumstance, all the active constituents of the interfered wave would have been completely eroded and the resulting wave which is now parasitically monochromatic, will eventually attenuate to zero, since the 'parasitic wave' does not have its own independent parameters for sustaining a continuous existence [10].

Any actively defined physical system carries along with it an inbuilt attenuating factor such that even in the absence of any external influence the system will eventually come to rest after a specified time. This accounts for the non-permanent nature of any physical system. A 'parasitic wave' as the name implies, has the ability of destroying or transforming the intrinsic constituents of the 'host wave' to its form after a sufficiently long time. It contains an inbuilt multiplier λ which is capable of raising the intrinsic parameters of the 'parasitic wave' to become equal to those of the 'host wave'. Consequently, once this equality is achieved, then all the active components of the host wave would have been completely eroded and it ceases to exist.

Fourier series has long provided one of the principal methods of analysis for mathematical physics, engineering, and signal processing. It has spurred generalizations and applications that continue to develop right up to the present. While the original theory of Fourier series applies to periodic functions occurring in wave motion, such as with light and sound, its generalizations often relate to wider settings, such as the time-frequency analysis underlying the recent theories of wavelet analysis and local trigonometric analysis. Periodic functions arise in the study of wave motion, when a basic waveform repeats itself periodically [11].

In qualitative analysis, unlike numerical work, the number one is a fundamental number, an indiscriminate constant value which can only describe the neutral behaviour of a system of varying series. In consequence, the exact behaviour of a non-stationary system may not be studied in the indiscriminate region of a constant value. Thus the constant value term which is a non-zero-order approximation may therefore be neglected from the varying series solution by the 'second world approximation' or the 'third world approximation'. Thus the approximation has the advantage of fast convergence of result and high degree of minimization. It also helps to control the complex anomalous behaviour of any possible square root displacement function which may produce imaginary result [10].

This paper is outlined as follows. Section 1, illustrates the basic concept of the work under study. The mathematical theory is presented in section 2. The results obtained are shown in section 3. While in section 4, we present the analytical discussion of the results obtained. The conclusion of this work is shown in section 5. This is immediately followed by appendix of some useful identities and a list of references.

1.1 Research Methodology

In this work we used Fourier analysis to investigate the behaviour of a carrier wave propagating in a 2D free space. A carrier wave as the name implies is the resultant interference of a 'parasitic wave' on a 'host wave'. The 'second world approximation' was first utilized to minimize the oscillating amplitude of the carrier wave**.** The oscillating amplitude and the spatial oscillating phase which were determined separately by Fourier analysis are convoluted to obtain the velocity with which the CW is propagating.

2.0 Mathematical Theory

2.1 Dynamical Theory of Superposition of Two Incoherent Waves.

The interference of one wave y_2 say 'parasitic wave' on another one y_1 say 'host wave' could cause the 'host wave' to decay to zero if they are out of phase. The decay process of y_1 can be gradual, over-damped or critically damped

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depending on the rate in which the amplitude of the host wave is brought to zero. However, the general concept is that the combination of y_1 and y_2 would first yield a third stage called the resultant wave say y, before the process of decay sets in. In this work, we refer to the resultant wave as the constituted carrier wave CCW. Now let us consider two incoherent waves defined by the displacement vectors

$$y_1(\vec{r},t) = a\beta\cos\left(\vec{k}\beta.\vec{r} - n\beta t - \varepsilon\,\beta\right) \qquad (2.1)$$

$$y_2(\vec{r},t) = b\lambda\cos\left(\vec{k}'\lambda.\vec{r} - n'\lambda t - \varepsilon'\lambda\right) \qquad (2.2)$$

where all the symbols have their usual wave related meaning. In this study, (2.1) is regarded as the 'host wave' whose propagation depends on the inbuilt multiplier β (= 0, 1, 2, . . . $\beta_{\max}$). While (2.2) represents a 'parasitic wave' also with an inbuilt multiplier λ (= 0, 1, 2, . . . $\lambda_{\max}$). The inbuilt multipliers are both dimensionless and as the name implies, they have the ability of gradually raising the basic intrinsic parameters of both waves respectively with time.

We have already established in a previous paper [12] that when (2.2) is superposed on (2.1) we get that

$$y = \left\{(a^2 - b^2\lambda^2) - 2\left(a - b\lambda\right)^2\cos\left((n-n'\lambda)t - (\varepsilon - \varepsilon'\lambda\right)\right\}^{\frac{1}{2}}$$
$$\cos\left(\vec{k}_c.\vec{r} - (n-n'\lambda)t - E\right) \qquad (2.3)$$

Equation (2.3) is regarded as the carrier wave (CW) and it is the equation that governs the dynamical behaviour of the coexistence of the HIV parasite in the human blood circulating system. It is obvious from the equation that once the characteristics of the 'parasitic wave' become equal to those of the 'host wave' as a result of the multiplier λ, then the CW goes to zero and the 'host wave' ceases to exist. Here we assume $\beta = 1$, a constant in this work and leave its variation for future study.

By interpretation E represents the total phase angle of the CW and $\vec{k}_c.\vec{r} = (k - k'\lambda)\,r\,(\cos\varphi + \sin\varphi)$, is a two dimensional (2D) coordinate position vector. By definition: $(n - n'\lambda)$ is the modulation angular frequency, $(k - k'\lambda)$ is the modulation propagation constant, the phase difference δ between the two interfering waves is $(\varepsilon - \varepsilon'\lambda)$ while $2\left(a - b\lambda\right)^2\cos\left((n-n'\lambda)t - (\varepsilon - \varepsilon'\lambda\right)$ is the interference term. Waves out of phase interfere destructively according to $\left(a - b\lambda\right)^2$ and wave's in-phase interferes constructively according to $\left(a + b\lambda\right)^2$.

In the regions where the amplitude of the wave is greater than either of the amplitude of the individual wave, we have constructive interference that means the path difference is $(\varepsilon + \varepsilon'\lambda)$, otherwise, it is destructive in which case the path difference is $(\varepsilon - \varepsilon'\lambda)$. If $n = n'$, then the average angular frequency say $(n + n'\lambda)/2$ will be much more greater than the modulation angular frequency say $(n - n'\lambda)/2$ and once this is achieved then we will have a slowly varying carrier wave with a rapidly oscillating phase. The total phase angle and its variation with respect to time give the characteristic angular velocity $Z(t)$. That is

$$E = \tan^{-1}\left(\frac{a\sin\varepsilon + b\lambda\sin\left(\varepsilon'\lambda - (n-n'\lambda)t\right)}{a\cos\varepsilon + b\lambda\cos\left(\varepsilon'\lambda - (n-n'\lambda)t\right)}\right) \qquad (2.4)$$

$$\frac{dE}{dt} = -Z(t)$$
$$= -\left((n-n'\lambda)\right)\left(\frac{b^2\lambda^2 + ab\lambda\cos\left((\varepsilon - \varepsilon'\lambda) + (n-n'\lambda)t\right)}{a^2 + b^2\lambda^2 + 2ab\lambda\cos\left((\varepsilon - \varepsilon'\lambda) + (n-n'\lambda)t\right)}\right)$$

(2.5)

However, it should be observed that in the absence of the multiplier $(\lambda = 0)$ the characteristic angular velocity does not exist. Now let us decompose the carrier wave equation CW into two functions; function of the oscillating amplitude $f(A)$ and the function of the spatial oscillating phase $f(\theta)$. Hence

$$y_{\max} = f(A) = \left\{(a^2 - b^2\lambda^2) - 2\left(a - b\lambda\right)^2\cos\left((n-n'\lambda)t - (\varepsilon - \varepsilon'\lambda\right)\right\}^{\frac{1}{2}}$$

(2.6)

$$f(\theta) = \cos\left(\vec{k}_c.\vec{r} - (n-n'\lambda)t - E\right) \qquad (2.7)$$

Equation (2.6) also represents the maximum displacement of the CW since the amplitude is a maximum when the spatial oscillating phase is assumed equal to one or ignored.

2.2 Differentio-Binomial Expansion or the 'Second World Approximation'.

It will not be very easy to expand (2.6) using Fourier series. As a result, there is need for us to obtain a comprehensively valid approximate solution to it before expanding it in

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Fourier series. Hence to make it valid for the application of Fourier series expansion, we first minimize it using Binomial expansion and thereafter the resulting equation is differentiated with respect to the variable time. However, if we differentiate the result of the Binomial expansion with respect to time, the resulting oscillating amplitude will be converted from the usual dimension of length which is meters (m) to angular velocity whose unit is radian per second (rad./s). We can however, further rearrange (2.6) for the purpose of the approximation as

$$f(A) = \sqrt{(a^2 - b^2\lambda^2)}\left\{1 - \frac{2(a-b\lambda)^2}{(a^2-b^2\lambda^2)}\cos\left((n-n'\lambda)t - (\varepsilon-\varepsilon'\lambda)\right)\right\}^{\frac{1}{2}}$$
(2.8)

$$f(A) = \sqrt{(a^2 - b^2\lambda^2)}\,\frac{d}{dt}\left\{1 - \frac{(a-b\lambda)^2}{(a^2-b^2\lambda^2)}\cos\left((n-n'\lambda)t - (\varepsilon-\varepsilon'\lambda)\right) + \ldots\right\}$$
(2.9)

$$f(A) = \sqrt{(a^2 - b^2\lambda^2)}\left\{\frac{(a-b\lambda)^2(n-n'\lambda)}{(a^2-b^2\lambda^2)}\sin\left((n-n'\lambda)t - (\varepsilon-\varepsilon'\lambda)\right) + \ldots\right\}$$
(2.10)

$$f(A) = \left\{D(n-n'\lambda)\sin\left((n-n'\lambda)t - (\varepsilon-\varepsilon'\lambda)\right)\right\}$$
(2.11)

Thus for clarity of purpose we have set

$$D = (a-b\lambda)^2 / \sqrt{(a^2-b^2\lambda^2)}\,. \qquad (2.13)$$

2.3 Fourier Series Expansion of the Oscillating Amplitude $f(A)$ of the CW.

The cornerstone of Fourier theory is a theorem which states that almost any periodic function can be analyzed into a series of harmonic functions with periods τ, $\tau/2$, $\tau/3$, ... , where τ is the period of the function under analysis [13]. Expansion of an oscillating function by Fourier series gives all modes of oscillation (fundamental and all overtones) which is extremely useful in physics. In particular, astronomical phenomena are usually periodic, as are animal heartbeats, tides and vibrating strings, so it makes sense to express them in terms of periodic functions. Now, by expanding the oscillating term of (2.11) in terms of Fourier series we get

$$F\left[f(A)\right]$$
$$= C_0 + C_1\{\sin((n-n'\lambda)t - (\varepsilon-\varepsilon'\lambda))\} + C_2\{\sin(2(n-n'\lambda)t - (\varepsilon-\varepsilon'\lambda))\}$$
$$+ C_3\{\sin(3(n-n'\lambda)t - (\varepsilon-\varepsilon'\lambda))\} + \ldots + C_\beta\{\sin(\beta(n-n'\lambda)t - (\varepsilon-\varepsilon'\lambda))\}$$
(2.14)

$$F\left[f(A)\right] = C_0 + \sum_{\beta=1}^{\infty} C_\beta\left\{\sin\left(\beta(n-n'\lambda)t - (\varepsilon-\varepsilon'\lambda)\right)\right\}$$
(2.15)

Thus (2.15) represents the Fourier series expansion of the oscillating amplitude for only one phase described by the sine (odd) function. It is however not always convenient to specify amplitude and phase [14] we can express each term in the form

$$C_\beta\left\{\sin\left(\beta(n-n'\lambda)t - (\varepsilon-\varepsilon'\lambda)_\beta\right)\right\} = A_\beta\cos\beta\left((n-n'\lambda)t\right) + B_\beta\sin\beta\left((n-n'\lambda)t\right)$$
(2.16)

where

$$\left.\begin{aligned} A_\beta &= C_\beta\cos(\varepsilon-\varepsilon'\lambda) \\ B_\beta &= -C_\beta\cos(\varepsilon-\varepsilon'\lambda) \end{aligned}\right\} \Rightarrow \qquad C_\beta = \sqrt{A_\beta^2 + B_\beta^2}$$
(2.17)

The negative sign indicates complex conjugate of the real part and the inclusions will make the dynamic components of the phase angle real. Thus (2.17) represents the amplitude of the nth harmonic. Where β is the Fourier index. From (2.16) if $\beta = 0$ then;

$$C_0\left\{\sin\left(-(\varepsilon-\varepsilon'\lambda)\right)\right\} = A_0 \quad \Rightarrow$$

$$C_0 = -\frac{A_0}{\sin(\varepsilon-\varepsilon'\lambda)} \qquad (2.18)$$

Thus the series expansion given by (2.15) can be rewritten using (2.16) as

$$F\left[f(A)\right] =$$
$$C_0 + \sum_{\beta=1}^{\infty}\left\{A_\beta\cos\beta\left((n-n'\lambda)t\right) + B_\beta\sin\beta\left((n-n'\lambda)t\right)\right\}$$
(2.19)

By definition A_0, A_β and B_β are the Fourier coefficients of the series expansion of the CWE. Thus (2.19) represents simultaneously the Fourier series expansion for both the cosine (even) and sine (odd) functions. The equations (2.15) or (2.19) can be appropriately used to study 2D wave interference in the Fourier series representation. However, in this work we are going to utilize (2.15).

2.4 Determination of the Fourier Coefficients of the Fourier Series Expansion.

The Fourier components of $F\left[f(A)\right]$ in (2.15) and (2.19) are given by the Euler formulas

$$A_0 = \frac{1}{\tau}\int_0^\tau f(A)\,dt$$

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$$= \frac{1}{\tau}\int_0^{\tau}\left[D(n-n'\lambda)\sin\left((n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\right)\right]dt$$
(2.20)

$$A_\beta = \frac{1}{\tau}\int_0^{\tau} f(A)\cos\beta\left((n-n'\lambda)t\right)dt$$

$$= \frac{1}{\tau}\int_0^{\tau}\left\{D\sin\left((n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\right)\right\}\cos\beta\left((n-n'\lambda)t\right)dt$$
(2.21)

$$B_\beta = \frac{1}{\tau}\int_0^{\tau} f(A)\sin\beta\left((n-n'\lambda)t\right)dt$$

$$= \frac{1}{\tau}\int_0^{\tau}\left\{D(n-n'\lambda)\sin\left((n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\right)\right\}\sin\beta\left((n-n'\lambda)t\right)dt$$
(2.22)

$$A_0 = -\frac{D}{\tau}\left\{\cos\left((n-n'\lambda)\tau-(\varepsilon-\varepsilon'\lambda)\right)-\cos\left(-(\varepsilon-\varepsilon'\lambda)\right)\right\}$$
(2.23)

$$A_0 = \frac{(a-b\lambda)^2(n-n'\lambda)}{2\pi\sqrt{(a^2-b^2\lambda^2)}}\left\{\cos\left(2\pi-(\varepsilon-\varepsilon'\lambda)\right)-\cos\left((\varepsilon-\varepsilon'\lambda)\right)\right\}$$
(2.24)

$$A_0 = -\frac{(a-b\lambda)^2(n-n'\lambda)\sin(\pi)\sin\left(\pi-(\varepsilon-\varepsilon'\lambda)\right)}{\pi\sqrt{(a^2-b^2\lambda^2)}}$$
(2.25)

$$C_0 = \frac{(a-b\lambda)^2(n-n'\lambda)\sin(\pi)\sin\left(\pi-(\varepsilon-\varepsilon'\lambda)\right)}{\pi\sqrt{(a^2-b^2\lambda^2)}\ \sin(\varepsilon-\varepsilon'\lambda)}$$
(2.26)

This gives the dimension of C_0 as radian per second (rad./s) which is the unit of angular velocity. Please see the appendix for the identities we have used to get these results.

$$A_\beta = \frac{D(n-n'\lambda)}{2\tau}\left\{\int_0^{\tau}\sin\left((1+\beta)(n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\right)+\int_0^{\tau}\sin\left((1-\beta)(n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\right)\right\}dt \qquad (2.27)$$

$$A_\beta = -\frac{D(n-n'\lambda)}{2\tau}\left\{\frac{\cos\left((1+\beta)(n-n'\lambda)\tau-(\varepsilon-\varepsilon'\lambda)\right)-\cos\left(-(\varepsilon-\varepsilon'\lambda)\right)}{(1+\beta)(n-n'\lambda)}\right\} - \frac{D(n-n'\lambda)}{2\tau}\left\{\frac{\cos\left((1-\beta)(n-n'\lambda)\tau-(\varepsilon-\varepsilon'\lambda)\right)-\cos\left(-(\varepsilon-\varepsilon'\lambda)\right)}{(1-\beta)(n-n'\lambda)}\right\}$$
(2.28)

The second term on the right side of (2.28) is ignored since if $\beta = 1$ according to the summation rule the expression in the parenthesis is infinite and will not be useful.

$$A_\beta = -\frac{(a-b\lambda)^2(n-n'\lambda)}{4\pi\sqrt{(a^2-b^2\lambda^2)}}\left\{\frac{\cos\left(2(1+\beta)\pi-(\varepsilon-\varepsilon'\lambda)\right)-\cos\left((\varepsilon-\varepsilon'\lambda)\right)}{(1+\beta)}\right\}$$
(2.29)

$$A_\beta = \frac{(a-b\lambda)^2(n-n'\lambda)}{2\pi\sqrt{(a^2-b^2\lambda^2)}}\left\{\frac{\sin\left((1+\beta)\pi\right)\sin\left((1+\beta)\pi-(\varepsilon-\varepsilon'\lambda)\right)}{(1+\beta)}\right\}$$
(2.30)

Finally, we have for B_β that

$$B_\beta = \frac{D(n-n'\lambda)}{2\tau}\left\{\int_0^{\tau}\cos\left((1-\beta)(n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\right)-\int_0^{\tau}\cos\left((1+\beta)(n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\right)\right\}dt \qquad (2.31)$$

$$B_\beta = \frac{D(n-n'\lambda)}{2\tau}\left\{\frac{\sin\left((1-\beta)(n-n'\lambda)\tau-(\varepsilon-\varepsilon'\lambda)\right)-\sin\left(-(\varepsilon-\varepsilon'\lambda)\right)}{(1-\beta)(n-n'\lambda)}\right\} - \frac{D(n-n'\lambda)}{2\tau}\left\{\frac{\sin\left((1+\beta)(n-n'\lambda)\tau-(\varepsilon-\varepsilon'\lambda)\right)-\sin\left(-(\varepsilon-\varepsilon'\lambda)\right)}{(1+\beta)(n-n'\lambda)}\right\}$$
(2.32)

The first term on the right side of (2.32) is also ignored based on the previous argument. Hence

$$B_\beta = -\frac{(a-b\lambda)^2(n-n'\lambda)}{4\pi\sqrt{(a^2-b^2\lambda^2)}}\left\{\frac{\sin\left(2(1+\beta)\pi-(\varepsilon-\varepsilon'\lambda)\right)+\sin\left((\varepsilon-\varepsilon'\lambda)\right)}{(1+\beta)}\right\}$$
(2.33)

$$B_\beta = -\frac{(a-b\lambda)^2(n-n'\lambda)}{2\pi\sqrt{(a^2-b^2\lambda^2)}}\left\{\frac{\sin\big((1+\beta)\pi\big)\cos\big((1+\beta)\pi-(\varepsilon-\varepsilon'\lambda)\big)}{(1+\beta)}\right\}$$

(2.34)

Upon the substitution of (2.30) and (2.34) into (2.17) we get after careful simplification

$$C_\beta^2 = \left(\frac{(a-b\lambda)^2(n-n'\lambda)}{2\pi\sqrt{(a^2-b^2\lambda^2)}}\right)^2\left(\frac{\sin^2(1+\beta)\pi}{(1+\beta)^2}\right) \Rightarrow C_\beta =$$

$$\left(\frac{(a-b\lambda)^2(n-n'\lambda)}{2\pi\sqrt{(a^2-b^2\lambda^2)}}\right)\left(\frac{\sin(1+\beta)\pi}{(1+\beta)}\right) \qquad (2.35)$$

Finally, upon the substitution of (2.26) and (2.35) into (2.15) we realize

$$F\big[f(A)\big] =$$

$$\frac{(a-b\lambda)^2(n-n'\lambda)\sin(\pi)\sin\big(\pi-(\varepsilon-\varepsilon'\lambda)\big)}{\pi\sqrt{(a^2-b^2\lambda^2)}\ \sin(\varepsilon-\varepsilon'\lambda)} +$$

$$\left(\frac{(a-b\lambda)^2(n-n'\lambda)}{2\pi\sqrt{(a^2-b^2\lambda^2)}}\right)\sum_{\beta=1}^{\infty}\left(\frac{\sin(1+\beta)\pi}{(1+\beta)}\right)\sin\big(\beta(n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\big)$$

(2.36)

Thus (2.36) represents the Fourier transform of the velocity component of the oscillating amplitude of the CW.

2.5 Fourier Series Expansion of the Spatial Oscillating Phase $f(\theta)$ of the CWE.

Now that we have taken the Fourier transform of the oscillating amplitude we can now proceed to calculate the Fourier transform of the spatial oscillating phase with respect to the position vector. Now we expand (2.7) by Fourier series as

$$F\big[f(\theta)\big] =$$

$$C_0 + C_1\cos\big(\vec{k}_c.\vec{r}-(n-n'\lambda)t-E\big) + C_2\cos\big(2\vec{k}_c.\vec{r}-(n-n'\lambda)t-E\big) +$$

$$C_3\cos\big(3\vec{k}_c.\vec{r}-(n-n'\lambda)t-E\big) + \ldots + C_\beta\cos\big(\beta\vec{k}_c.\vec{r}-(n-n'\lambda)t-E\big)$$

(2.41)

$$F\big[f(\theta)\big] = C_0 + \sum_{\beta=1}^{\infty} C_\beta\cos\big(\beta\vec{k}_c.\vec{r}-(n-n'\lambda)t-E\big)$$

(2.42)

However, there is need to separate the function in the summation sign into two components.

$$C_\beta\cos\big(\beta\vec{k}_c.\vec{r}-(n-n'\lambda)t-E\big) = A_\beta\cos\big(\beta\vec{k}_c.\vec{r}\big) + B_\beta\sin\big(\beta\vec{k}_c.\vec{r}\big)$$

(2.43)

With the assumption that

$$A_\beta = C_\beta\cos\big(-(n-n'\lambda)t-E\big) \Rightarrow$$

$$A_\beta = C_\beta\cos\big((n-n'\lambda)t+E\big) \qquad (2.44)$$

$$B_\beta = -C_\beta\sin\big(-(n-n'\lambda)t-E\big) \Rightarrow$$

$$B_\beta = C_\beta\sin\big((n-n'\lambda)t+E\big) \qquad (2.45)$$

$$C_\beta = \sqrt{A_\beta^2+B_\beta^2} \qquad (2.46)$$

Also upon the substitution of (2.43) into (2.42) we get

$$F\big[f(\theta)\big] = C_0 + \sum_{\beta=1}^{\infty} A_\beta\cos\big(\beta\vec{k}_c.\vec{r}\big) + B_\beta\sin\big(\beta\vec{k}_c.\vec{r}\big)$$

(2.47)

From (2.43): if $\beta = 0 \Rightarrow$

$$C_0 = \frac{1}{\cos\big((n-n'\lambda)t+E\big)}A_0 \qquad (2.48)$$

2.6 Determination of the Fourier Coefficients of the Fourier Series Expansion.

The Fourier components of (2.47) A_0 , A_β and C_β are given by the Euler formulas

$$A_0 = \frac{1}{l}\int_0^l f(\theta)\,dr$$

$$= \frac{1}{l}\int_0^l \cos\big(\vec{k}_c.\vec{r}-(n-n'\lambda)t-E\big)dr \qquad (2.49)$$

$$A_\beta = \frac{2}{l}\int_0^l f(\theta)\cos\big(\beta\vec{k}_c.\vec{r}\big)dr$$

$$= \frac{2}{l}\int_0^l \cos\big(\vec{k}_c.\vec{r}-(n-n'\lambda)t-E\big)\cos\big(\beta\vec{k}_c.\vec{r}\big)dr \qquad (2.50)$$

$$B_\beta = \frac{2}{l}\int_0^l f(\theta)\sin\big(\beta\vec{k}_c.\vec{r}\big)dr$$

$$= \frac{2}{l}\int_0^l \cos\big(\vec{k}_c.\vec{r}-(n-n'\lambda)t-E\big)\sin\big(\beta\vec{k}_c.\vec{r}\big)dr$$

(2.51)

$$\vec{k}_c = (k-k'\lambda)\,i + (k-k'\lambda)\,j\ ;\ \vec{r} = r\,(\cos\varphi\, i + \sin\varphi\, j)$$

$$\Rightarrow \quad \vec{k}_c.\vec{r} = (k-k'\lambda)\,r\,(\cos\varphi+\sin\varphi) \qquad (2.52)$$

Note that we changed from Cartesian coordinate to 2D polar coordinate system.

$$A_0 = \frac{1}{l}\int_0^l \cos\left((k-k'\lambda)\, r\,(\cos\varphi+\sin\varphi)-(n-n'\lambda)\,t-E\right)dr$$

(2.53)

$$A_0 = \frac{\sin\left((k-k'\lambda)\; l\,(\cos\varphi+\sin\varphi)-(n-n'\lambda)\,t-E\right)-\sin\left(-(n-n'\lambda)\,t-E\right)}{(k-k'\lambda)\; l\,(\cos\varphi+\sin\varphi)}$$

(2.54)

$$A_0 = \frac{\sin\left((k-k'\lambda)\; l\,(\cos\varphi+\sin\varphi)-(n-n'\lambda)\,t-E\right)+\sin\left((n-n'\lambda)\,t+E\right)}{(k-k'\lambda)\; l\,(\cos\varphi+\sin\varphi)}$$

(2.55)

$$C_0 = \frac{\sin\left((k-k'\lambda)\; l\,(\cos\varphi+\sin\varphi)-(n-n'\lambda)\,t-E\right)+\sin\left((n-n'\lambda)\,t+E\right)}{(k-k'\lambda)\; l\,(\cos\varphi+\sin\varphi)\cos\left((n-n'\lambda)\,t+E\right)}$$

(2.56)

Also upon using the relation; $2\cos A\cos B = \cos(A+B)+\cos(A-B)$ then (2.50) will reduce to

$$A_\beta = \frac{2}{l}\int_0^l \frac{1}{2}\left\{\cos\left((k-k'\lambda)\,r\,(\cos\varphi+\sin\varphi)-(n-n'\lambda)\,t-E+\beta\left((k-k'\lambda)\,r\,(\cos\varphi+\sin\varphi)\right)\right)\right\}dr +$$

$$\frac{2}{l}\int_0^l \frac{1}{2}\left\{\cos\left((k-k'\lambda)\,r\,(\cos\varphi+\sin\varphi)-(n-n'\lambda)\,t-E-\beta\left((k-k'\lambda)\,r\,(\cos\varphi+\sin\varphi)\right)\right)\right\}dr$$

(2.57)

$$A_\beta = \frac{2}{l}\int_0^l \frac{1}{2}\left\{\cos\left((1+\beta)(k-k'\lambda)\,r\,(\cos\varphi+\sin\varphi)-(n-n'\lambda)\,t-E\right)\right\}dr +$$

$$\frac{2}{l}\int_0^l \frac{1}{2}\left\{\cos\left((1-\beta)(k-k'\lambda)\,r\,(\cos\varphi+\sin\varphi)-(n-n'\lambda)\,t-E\right)\right\}dr$$

(2.58)

$$A_\beta = \frac{\left\{\sin\left((1+\beta)(k-k'\lambda)\; l\,(\cos\varphi+\sin\varphi)-(n-n'\lambda)\,t-E\right)-\sin\left(-(n-n'\lambda)\,t-E\right)\right\}}{(k-k'\lambda)\; l\,(\cos\varphi+\sin\varphi)(1+\beta)} +$$

$$\frac{\left\{\sin\left((1-\beta)(k-k'\lambda)\; l\,(\cos\varphi+\sin\varphi)-(n-n'\lambda)\,t-E\right)-\sin\left(-(n-n'\lambda)\,t-E\right)\right\}}{(k-k'\lambda)\; l\,(\cos\varphi+\sin\varphi)(1-\beta)}$$

(2.59)

The second term on the right side of (2.59) is ignored since the equation becomes infinite if $\beta = 1$. Hence

$$A_\beta = \left(\frac{\sin\left((1+\beta)(k-k'\lambda)\; l\,(\cos\varphi+\sin\varphi)-(n-n'\lambda)\,t-E\right)+\sin\left((n-n'\lambda)\,t+E\right)}{(k-k'\lambda)\; l\,(\cos\varphi+\sin\varphi)(1+\beta)}\right)$$

(2.60)

Finally, by following the same procedure that led to (2.63) we can solve for B_β in (2.54) to get

$$B_\beta = -\left(\frac{\cos\left((1+\beta)(k-k'\lambda)\; l\,(\cos\varphi+\sin\varphi)-(n-n'\lambda)\,t-E\right)-\cos\left((n-n'\lambda)\,t+E\right)}{(k-k'\lambda)\; l\,(\cos\varphi+\sin\varphi)(1+\beta)}\right)$$

(2.61)

Thus A_β and B_β are both dimensionless. We can now substitute (2.60) and (2.61) into (2.46) as

$$C_\beta^2 =$$

$$\left(\frac{\sin\left((1+\beta)(k-k'\lambda)\; l\,(\cos\varphi+\sin\varphi)-(n-n'\lambda)\,t-E\right)+\sin\left((n-n'\lambda)\,t+E\right)}{(k-k'\lambda)\; l\,(\cos\varphi+\sin\varphi)(1+\beta)}\right)^2$$

$$+$$

$$\left(\frac{\cos\left((n-n'\lambda)\,t+E\right)-\cos\left((1+\beta)(k-k'\lambda)\; l\,(\cos\varphi+\sin\varphi)-(n-n'\lambda)\,t-E\right)}{(k-k'\lambda)\; l\,(\cos\varphi+\sin\varphi)(1+\beta)}\right)^2$$

(2.62)

After a careful and lengthy algebra and with the application of trigonometric identity, see appendix, we realize

$$C_\beta = \left(\frac{\sqrt{2-2\cos\left((1+\beta)(k-k'\lambda)\; l\,(\cos\varphi+\sin\varphi)\right)}}{(k-k'\lambda)\; l\,(\cos\varphi+\sin\varphi)(1+\beta)}\right)$$

(2.63)

We can now replace (2.56) and (2.63) into (2.42) and simplify to get

$$F\left[f(\theta)\right] =$$

$$\frac{\sin\left((k-k'\lambda)\; l\,(\cos\varphi+\sin\varphi)-(n-n'\lambda)\,t-E\right)+\sin\left((n-n'\lambda)\,t+E\right)}{(k-k'\lambda)\; l\,(\cos\varphi+\sin\varphi)\cos\left((n-n'\lambda)\,t+E\right)}$$

$$+$$

$$\sum_{\beta=1}^{\infty}\left(\frac{\sqrt{2-2\cos\left((1+\beta)(k-k'\lambda)\; l\,(\cos\varphi+\sin\varphi)\right)}}{(k-k'\lambda)\; l\,(\cos\varphi+\sin\varphi)(1+\beta)}\right)\cos\left(\beta(k-k'\lambda)\,r\,(\cos\varphi+\sin\varphi)-(n-n'\lambda)t-E\right)$$

(2.64)

However it is also clear that in the absence of the 'parasitic wave' in which case $\lambda = 0$ we get

$$F\left[f(\theta)\right] =$$

$$\frac{\sin\left(k\;\, l\,(\cos\varphi+\sin\varphi)-n\,t-\left|\partial E\right|\right)+\sin\left(n\,t+E\right)}{k\;\, l\,(\cos\varphi+\sin\varphi)\cos\left(nt+E\right)} \quad +$$

$$\sum_{\beta=1}^{\infty}\left(\frac{\sqrt{2-2\cos\left((1+\beta)\,k\;\, l\,(\cos\varphi+\sin\varphi)\right)}}{k\;\, l\,(\cos\varphi+\sin\varphi)(1+\beta)}\right)\cos\left(\beta\,k\,r\,(\cos\varphi+\sin\varphi)-n\,t-E\right)$$

(2.65)

Thus $F\left[f(\theta)\right]$ is dimensionless. Note that: if $\lambda = 0$; $\vec{k}_c . \vec{r} = k\, r\,(\cos\varphi + \sin\varphi)$, $\varphi = \pi - \varepsilon$ and $E = \varepsilon$.

2.7 Convolution Theory of the Fourier Transform Of the Oscillating Amplitude $F\left[f(A)\right]$ and the Spatial Oscillating Phase $F\left[f(\theta)\right]$ of the CW.

Now that we have separately determined the Fourier series expansion of the oscillating amplitude $F\left[f(A)\right]$ and the spatial oscillating phase $F\left[f(\theta)\right]$ respectively. The necessary requirement now is to convolute them in order to obtain a concise equation of the CW. Convolution here means multiplying the oscillating amplitude and the spatial oscillating phase term by term. Let us represent the result of the convolution of these functions by H and then with the same displacement vector y which represents the CW.

$$v = H\left\{F\left[f(A)\right]; F\left[f(\theta)\right]\right\} \equiv F\left\{f(A)\right\} \otimes F\left\{f(\theta)\right\} \quad (2.66)$$

Also when we convolute (2.39) and (2.64) so that

$$v = H\left\{F\left[f(A)\right]; F\left[f(\theta)\right]\right\} = \frac{(a-b\lambda)^2 (n-n'\lambda)\left(\sin(\pi)\sin\left(\pi-(\varepsilon-\varepsilon'\lambda)\right)\right)}{\pi\sqrt{(a^2-b^2\lambda^2)}\ \sin(\varepsilon-\varepsilon'\lambda)} \times$$

$$\frac{\sin\left((k-k'\lambda)\ l\,(\cos\varphi+\sin\varphi)-(n-n'\lambda)t-E\right)+\sin\left((n-n'\lambda)t+E\right)}{(k-k'\lambda)\ l\,(\cos\varphi+\sin\varphi)\cos\left((n-n'\lambda)t+E\right)} +$$

$$\frac{(a-b\lambda)^2 (n-n'\lambda)\left(\sin(\pi)\sin\left(\pi-(\varepsilon-\varepsilon'\lambda)\right)\right)}{\pi\sqrt{(a^2-b^2\lambda^2)}\ \sin(\varepsilon-\varepsilon'\lambda)}$$

$$\sum_{\beta=1}^{\infty}\left(\frac{\sqrt{2-2\cos\left((1+\beta)(k-k'\lambda)\ l\,(\cos\varphi+\sin\varphi)\right)}}{(k-k'\lambda)\ l\,(\cos\varphi+\sin\varphi)(1+\beta)}\right)\times$$

$$\cos\left(\beta(k-k'\lambda)\, r\,(\cos\varphi+\sin\varphi)-(n-n'\lambda)t-\left|\partial E\right|\right)+$$

$$\left(\frac{(a-b\lambda)^2(n-n'\lambda)}{2\pi\sqrt{(a^2-b^2\lambda^2)}}\right)\sum_{\beta=1}^{\infty}\left(\frac{\sin\left((1+\beta)\pi\right)}{(1+\beta)}\right)\times$$

$$\frac{\sin\left((k-k'\lambda)\ l\,(\cos\varphi+\sin\varphi)-(n-n'\lambda)t-E\right)+\sin\left((n-n'\lambda)t+E\right)}{(k-k'\lambda)\ l\,(\cos\varphi+\sin\varphi)\cos\left((n-n'\lambda)t+E\right)}$$

$$\sin\left(\beta(n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\right) +$$

$$\left(\frac{(a-b\lambda)^2(n-n'\lambda)}{2\pi\sqrt{(a^2-b^2\lambda^2)}}\right)\sum_{\beta=1}^{\infty}\frac{\sin\left((1+\beta)\pi\right)}{(1+\beta)}$$

$$\left(\frac{\sqrt{2-2\cos\left((1+\beta)(k-k'\lambda)\ l\,(\cos\varphi+\sin\varphi)\right)}}{(k-k'\lambda)\ l\,(\cos\varphi+\sin\varphi)(1+\beta)}\right)\times$$

$$\sin\left(\beta(n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\right)$$

$$\cos\left(\beta(k-k'\lambda)\, r\,(\cos\varphi+\sin\varphi)-(n-n'\lambda)t-E\right)$$

(2.67)

In the absence of 'parasitic wave', that is when $\lambda = 0$, we realize the below equation.

$$v = H\left\{F\left[f(A)\right]; F\left[f(\theta)\right]\right\} = \frac{a\,n\ \sin(\pi)\sin\left(\pi-\varepsilon\right)}{\pi\ \sin(\varepsilon)}$$

$$\frac{\sin\left(k\,l\,(\cos\varphi+\sin\varphi)-n\,t-E\right)+\sin\left(n\,t+E\right)}{k\ l\,(\cos\varphi+\sin\varphi)\cos\left(n\,t+E\right)} +$$

$$\frac{a\,n\sin(\pi)\sin\left(\pi-\varepsilon\right)}{\pi\sin(\varepsilon-\varepsilon'\lambda)}$$

$$\sum_{\beta=1}^{\infty}\left(\frac{\sqrt{2-2\cos\left((1+\beta)\,k\ l\,(\cos\varphi+\sin\varphi)\right)}}{k\ l\,(\cos\varphi+\sin\varphi)(1+\beta)}\right)\cos\left(\beta(k\,r\,(\cos\varphi+\sin\varphi)-n\,t-E\right)+$$

$$\left(\frac{a\,n}{2\pi}\right)\frac{\sin\left(k\,l\,(\cos\varphi+\sin\varphi)-n\,t-E\right)+\sin\left(n\,t+E\right)}{k\,l\,(\cos\varphi+\sin\varphi)\cos\left(n\,t+E\right)}$$

$$\sum_{\beta=1}^{\infty}\frac{\sin\left((1+\beta)\pi\right)}{(1+\beta)}\sin\left(\beta(n\,t-\varepsilon)\right) +$$

$$\left(\frac{a\,n}{2\pi}\right)\sum_{\beta=1}^{\infty}\frac{\sin\left((1+\beta)\pi\right)}{(1+\beta)}$$

$$\left(\frac{\sqrt{2-2\cos\left((1+\beta)\,k\,l\,(\cos\varphi+\sin\varphi)\right)}}{k\ l\,(\cos\varphi+\sin\varphi)(1+\beta)}\right)$$

$$\sin\left(\beta(n\,t-\varepsilon)\right)\times$$

$$\cos\left(\beta(k\,r\,(\cos\varphi+\sin\varphi)-n\,t-E\right) \quad (2.68)$$

In this study we assume arbitrary values for the radius of the cylindrical coordinate pipe $r = 0.005m$ $(0.5m)$ and the wavelength λ of the CW as $l = 0.01m$ $(1cm)$. We also considered in this work only situation where the constraints are of equal weights, say $\alpha = \beta$. Otherwise, if we apply the double summation rule as it stands, that means, we shall first allow α take the value of one and let β run from one to infinity, again we allow α take the value of two and let β run from one to infinity and the process is repeated. However, since both constraints are of the same source function we can equate them so as to save us computation time and unnecessary difficult task.

2.7 Evaluation of the Relative Distance Covered by the Carrier wave CW in One Dimension (1D).

Suppose we want to evaluate the entire relative distance covered by the carrier wave as it propagates in one dimensional (1D) free linear space, then certain boundary conditions would have to be met. Under this circumstance, the carrier wave shall produce a corresponding distance for every value of the multiplier λ and the total distance covered would be the sum of these relative distances. Now let us consider the product differentiation of the oscillating amplitude and the spatial oscillatory phase as components of the CW.

$$v=\frac{dy}{dt}=(a-b\lambda)^2\,(n-n'\lambda)\sin\Big((n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\Big)\times$$

$$\Big((a^2-b^2\lambda^2)-2(a-b\lambda)^2\cos\Big((n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\Big)\Big)^{-\frac{1}{2}}\times\cos\Big((k-k'\lambda)x-(n-n'\lambda)t-E\Big)+$$

$$\Big((a^2-b^2\lambda^2)-2(a-b\lambda)^2\cos\Big((n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\Big)\Big)^{\frac{1}{2}}\times$$

$$\left((n-n'\lambda)+\frac{dE}{dt}\right)\sin\Big((k-k'\lambda)x-(n-n'\lambda)t-E\Big)\qquad(2.69)$$

It is assumed in this study that after a sufficiently long time and a specific distance covered, the carrier wave ceases to exist, that is, $y\to 0$ as $\lambda\to\lambda_{\max}$. Consequently, the velocity of the carrier wave must also tend to zero at the critical value $\lambda_{\max}$, hence, $v=dy/dt=0$, and

$$(a-b\lambda)^2\,(n-n'\lambda)\sin\Big((n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\Big)\times$$
$$\cos\Big((k-k'\lambda)x-(n-n'\lambda)t-E\Big)\times$$

$$\Big((a^2-b^2\lambda^2)-2(a-b\lambda)^2\cos\Big((n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\Big)\Big)^{-\frac{1}{2}}=$$

$$-\left((n-n'\lambda)+\frac{dE}{dt}\right)\sin\Big((k-k'\lambda)x-(n-n'\lambda)t-E\Big)\times$$

$$\Big((a^2-b^2\lambda^2)-2(a-b\lambda)^2\cos\Big((n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\Big)\Big)^{\frac{1}{2}}$$
(2.70)

Further division and rearrangement of (2.70) with the hope to produce a better result yields

$$(a-b\lambda)^2(n-n'\lambda)\sin\Big((n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\Big)\times$$

$$\Big(a^2-b^2\lambda^2\Big)^{-\frac{1}{2}}\left(1-\frac{2(a-b\lambda)^2}{(a^2-b^2\lambda^2)}\cos\Big((n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\Big)\right)^{-\frac{1}{2}}=$$

$$-\left((n-n'\lambda)+\frac{dE}{dt}\right)\tan\Big((k-k'\lambda)x-(n-n'\lambda)t-E\Big)\times$$

$$\Big(a^2-b^2\lambda^2\Big)^{\frac{1}{2}}\left(1-\frac{2(a-b\lambda)^2}{(a^2-b^2\lambda^2)}\cos\Big((n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\Big)\right)^{\frac{1}{2}}$$
(2.71)

In qualitative analysis, unlike numerical work, the number one is a fundamental number, an indiscriminate constant value which can only describe the neutral behaviour of a system of varying series. In consequence, the exact behaviour of a non-stationary system may not be studied in the indiscriminate region of a constant value. Thus the constant value term which is a non-zero-order approximation may therefore be neglected from the varying series solution by direct differentiation of the resulting Binomial equation.

We shall at this stage adopt a new form of approximation technique the "Differentio-Binomial" approximation. This approximation makes use of the second term in the series. The approximation has the advantage of fast convergence and high degree of minimization. The "Differentio-Binomial" approximation is defined as follows.

$$\big(1-x\big)^n=\frac{d}{dx}\left(1-nx-\frac{n(n-1)x^2}{2!}-\frac{n(n-1)(n-2)x^3}{3!}-\cdots\right)$$
(2.72)

Thus when we utilize this approximation on both sides of (2.71), we get after some simplification

$$-(a-b\lambda)^2(n-n'\lambda)(n-n'\lambda)\sin\Big((n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\Big)\times$$

$$\Big(a^2-b^2\lambda^2\Big)^{-\frac{1}{2}}\left(\frac{(a-b\lambda)^2}{(a^2-b^2\lambda^2)}\right)\sin\Big((n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\Big)=$$

$$-\left((n-n'\lambda)+\frac{dE}{dt} \right) \tan\left((k-k'\lambda)x-(n-n'\lambda)t-E\right)\times$$

$$(n-n'\lambda)\left(a^2-b^2\lambda^2\right)^{\frac{1}{2}}\left(\frac{(a-b\lambda)^2}{(a^2-b^2\lambda^2)}\right)\sin\left((n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\right)$$

(2.73)

$$\frac{(a-b\lambda)^2(n-n'\lambda)}{(a^2-b^2\lambda^2)}\sin\left((n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\right)=$$

$$\left((n-n'\lambda)-Z\right)\tan\left((k-k'\lambda)x-(n-n'\lambda)t-E\right) \quad (2.74)$$

$$x=\frac{1}{(k-k'\lambda)}\left\{\tan^{-1}\left(\frac{(a-b\lambda)^2(n-n'\lambda)}{(a^2-b^2\lambda^2)((n-n'\lambda)-Z)}\sin\left((n-n'\lambda)t-(\varepsilon-\varepsilon'\lambda)\right)\right)+(n-n'\lambda)t+E\right\}$$

(2.75)

In the absence of the multiplier $\lambda=0,\ Z=0, E=\varepsilon$, the relative distances covered by the CW is given by

$$x=\frac{1}{k}\left\{\tan^{-1}\left(\sin(nt-\varepsilon)\right)+nt+\varepsilon\right\} \quad (2.76)$$

Thus (2.75) and (2.76) are used to calculate the relative linear distances x covered by the carrier wave CW for each value of the multiplier $\lambda = 0, 1, \ldots, 43.63$. Note that while (2.75) depends on t and λ, (2.76) depend entirely on t. The graphs emanating from (2.75) and (2.76) are shown in fig. 3.9 of section 3.

2.8 Calculation of the unknown characteristics of the 'parasitic wave' from the known characteristics of the 'host wave'.

Let us now consider some arbitrary values of the characteristics of the 'host wave' contained in the carrier wave given by (2.3). Hence, let us assume that the 'host wave' has the following characteristics: a = 0.002 m, $n = 5$ rad. /s, ε =0.01746 rad., and k =1.7456 rad. /m. Then the assumption here is that after a prolonged damping time, the carrier wave $y \to 0$ as $\lambda \to \lambda_{\max}$, then the characteristics of both interfering waves become equal to one another and the carrier wave function becomes zero. Based on this simple argument we get the following relations.

$$a-b\lambda=0 \quad \Rightarrow \quad a=b\lambda \Rightarrow \quad 0.002=b\lambda$$
(2.77)

$$n-n'\lambda=0 \quad \Rightarrow \quad n=n'\lambda \quad \Rightarrow \quad 5=n'\lambda$$
(2.78)

$$\varepsilon-\varepsilon'\lambda=0 \quad \Rightarrow \quad \varepsilon=\varepsilon'\lambda \quad \Rightarrow \quad 0.01746=\varepsilon'\lambda$$
(2.79)

$$k-k'\lambda=0 \quad \Rightarrow \quad k=k'\lambda \quad \Rightarrow \quad 1.7456=k'\lambda$$
(2.80)

When we divide (2.77) by (2.78); (2.77) by (2.79); (2.78) by (2.79), and finally (2.79) by (2.80), we obtain respectively

$$0.0004n'=b\,; \quad 0.1146\varepsilon'=b\,; \quad 286.37\varepsilon'=n' \quad ; \quad 0.01k'=\varepsilon'$$
(2.81)

With the help of simple ratio the basic characteristics of the 'parasitic wave' can be found from (2.81) as:

b =0.00004584 m ; n' = 0.1146 rad. /s ; ε' =0.0004 rad. and k' = 0.04 rad. /m (2.82)

By using these basic characteristic values in any of (2.77) - (2.80), we generally obtain $\lambda_{\max}$ = 43.63. Thus, the physical range of interest of the inbuilt raising multiplier is, $0 \le \lambda \le 44$ where λ (= 0, 1, 2, … , 43, 43.63). We note that at the critical or maximum value of the multiplier $\lambda_{\max}$ all the characteristic values of the 'parasitic wave' contained in the carrier wave equation would have been correspondingly raised to become almost equal to those of the 'host wave'.

In consequence, we have succeeded in using the available known values of the characteristics of the 'host wave' in the carrier wave to determine the characteristic values of the 'parasitic wave' which were initially not known. Also these characteristic values are used to calculate the maximum value of the raising multiplier $\lambda_{\max}$ and hence its subsequent values are determined. The variation of the multiplier is choice dependent but we adopted a slow varying multiplier in such a way that we can understand clearly the physical parameter space which is assessable to the model that we have developed.

2.9 Determination of the Attenuation Constant (η).

Attenuation is a decay process. It brings about a gradual reduction and weakening in the initial strength of the intrinsic parameters of a given active system. In this study, the parameters are the amplitude (a), phase angle (ε), angular frequency (n) and the spatial frequency (k). The dimension of the attenuation constant (η) is determined by the system under study. However, in this work, attenuation constant is the relative rate of fractional change (FC) in the basic parameters of the carrier wave function. There are 4 (four) basic attenuating parameters present in the carrier wave function. Hence,

Average FC,

$$\sigma=\frac{1}{4}\times\left\{\left(\frac{a-b\lambda}{a}\right)+\left(\frac{n-n'\lambda}{n}\right)+\left(\frac{\varepsilon-\varepsilon'\lambda}{\varepsilon}\right)+\left(\frac{k-k'\lambda}{k}\right)\right\}$$
(2.83)

Attenuation constant,

$$\eta=\frac{FC\big|_{\lambda=i}-FC\big|_{\lambda=i+1}}{unit\ time\ (s)}=\frac{\sigma_i-\sigma_{i+1}}{unit\ time\ (s)}$$
(2.84)

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And its dimension is per second (s^{-1}). Thus (2.84) gives $0.022916s^{-1}$ for all values of i =1, 2, … , 43, 43.63.

2.10 Determination of the Decay Time (t) of the Carrier Wave CW.

We used the information provided in section 2.9, to compute the various times taken for the carrier wave to decay to zero. This is possible provided the value of the time when the raising multiplier is exactly one is known, that is, about the time when λ starts counting. The maximum time the carrier wave lasted as a function of the raising multiplier λ is also determined with the use of the attenuation equation shown in (2.84). The reader should note that we have adopted a slowly varying regular interval for the raising multiplier λ =0, 1, 2, …, 43, 43.63) for our study. The varying interval we adopt will help to delineate clearly the physical parameter space accessible to our model. However, it is clear from the calculation that the different attenuating fractional changes contained in the carrier wave function are approximately equal to one another. We can now apply the attenuation equation given below.

$$\sigma = e^{-2\eta t/\lambda} \qquad (2.85)$$

$$t = \frac{\lambda}{2\eta}\ln\sigma \qquad (2.86)$$

Clearly, we used (2.86) to calculate the exact value of the time corresponding to any value of the multiplier λ .

We used table scientific calculator and Microsoft excel to compute our results. Also the GNUPLOT 3.7 version was used to plot the corresponding graphs.

3.0 Presentation of Results

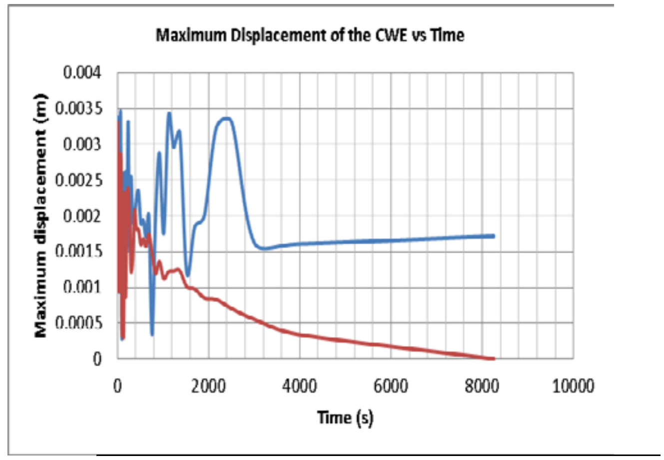

Fig. 3.1: (a) the upper blue curve which represents the maximum displacement when the multiplier $\lambda = 0$ and time [0, 8251], β =43.63 and (b) the lower brown curve which represents the maximum displacement when the multiplier λ [0, 43.63] and time [0, 8251s], β =43.63.

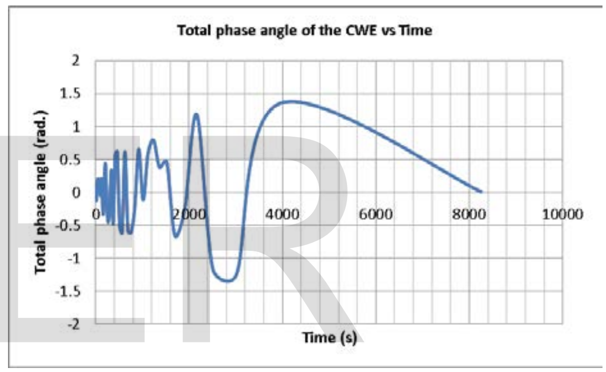

Fig. 3.2: Represents the multiplier λ [0, 43.63] and time [0, 8251s], β =43.63.

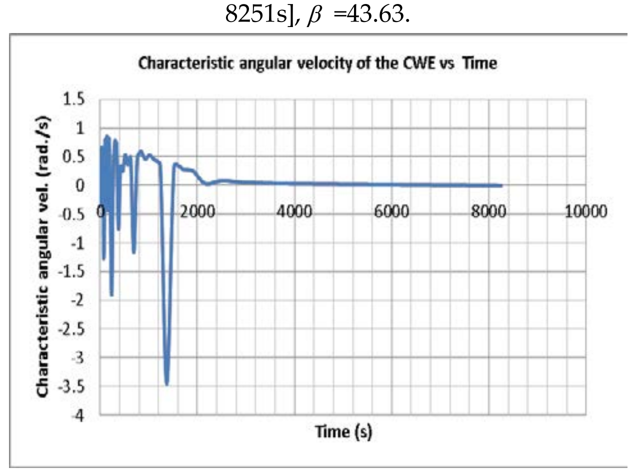

Fig. 3.3: Represents the multiplier λ [0, 43.63] and time [0, 8251s], β =43.63.

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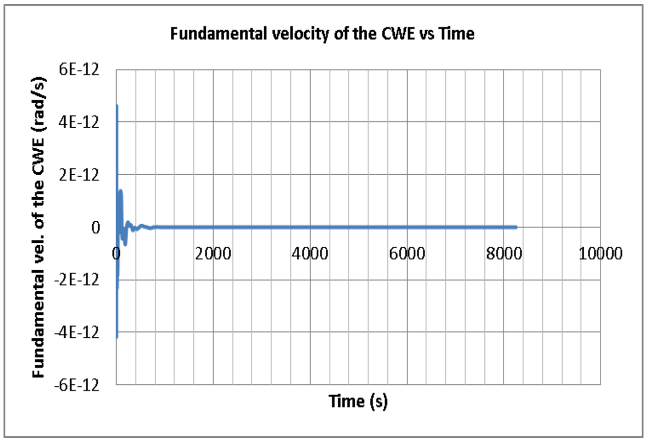

Fig. 3.4: Represents the multiplier λ = [0, 43.63] and time [0, 8251s] $\beta = 0$.

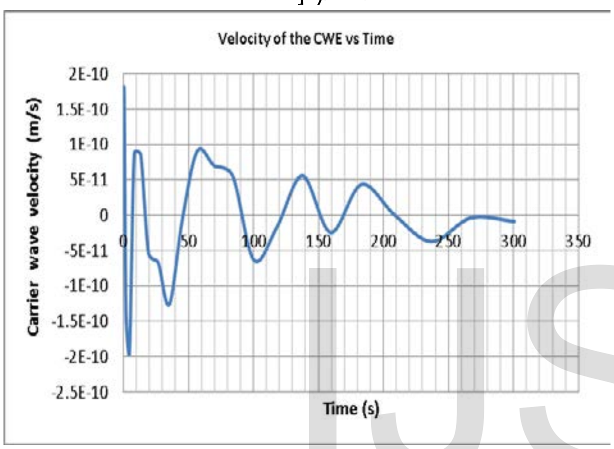

Fig. 3.5: Represents the multiplier λ = [0, 22] and time [0, 300s] $\beta = 43.63$.

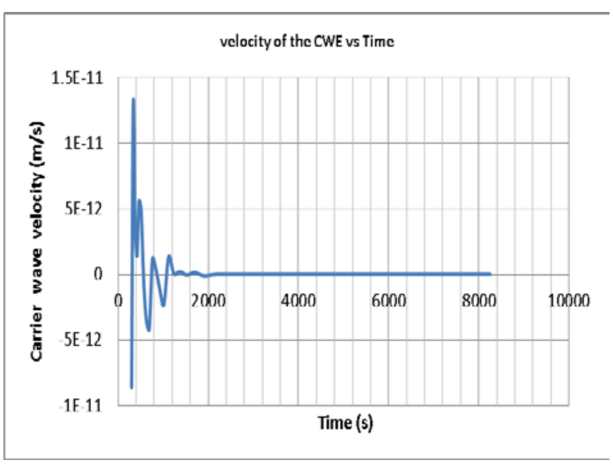

Fig. 3.6: Represents the multiplier λ = [22, 43.63] and time [300, 8251s] $\beta = 43.63$.

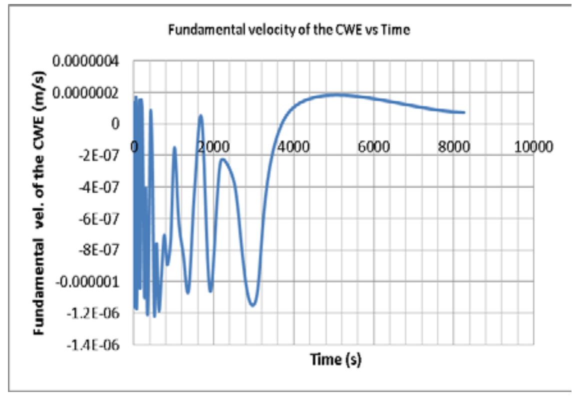

Fig. 3.7: Represents the multiplier $\lambda = 0$ and time [0, 8251s] $\beta = 0$, $Z = 0$, $E = \varepsilon$.

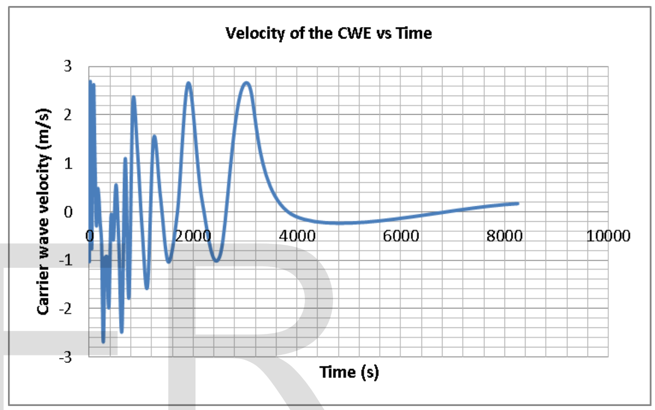

Fig. 3.8: Represents the multiplier $\lambda = 0$ and time [0, 8251s] $\beta = 43.63$, $Z = 0$, $E = \varepsilon$.

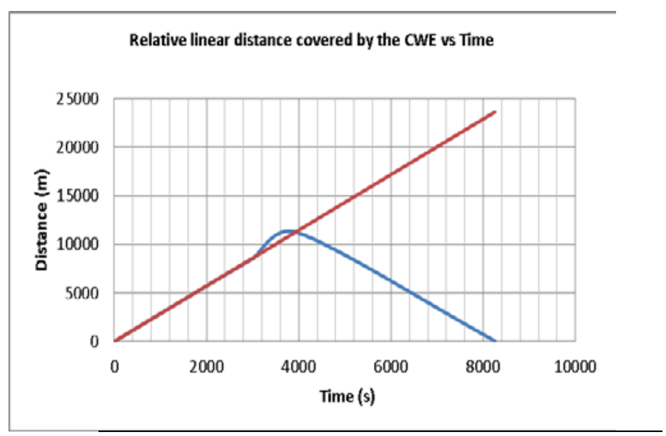

Fig. 3.9: Represents, (a) the brown straight line when the multiplier $\lambda = 0$ and time [0, 8251s], $\beta = 43.63$, $Z = 0$, $E = \varepsilon$ and (b) the blue straight and curved line when the multiplier λ = [0, 43.63] and time [0, 8251s], $\beta = 43.63$.

4.0 Discussion of Results

The relative attenuating parameters of the CW as they depend on the raising multiplier and time are shown in figs. 3.1 – 3.9. The graph of the total phase angle E, the characteristic angular velocity Z and the maximum displacement y_m which are given by (2.4), (2.5), and (2.6) is shown in figs. 3.1, 3.2 and 3.3. While the graph of the velocity gradient of the carrier wave CW which is given by (2.67) and (2.68) is represented by figs 3.4 – 3.9. Although, our work was confined to only when the Fourier index was 43.63, since we believe that this is the region of most relevant interest to our work. Note that figs. 3.4 and 3.7 which are the first term of equation (2.67) and (2.68) is the harmonic analysis of the CW and it does not contain the Fourier index β.

The decay process of the total phase angle, the characteristic angular velocity is not constant. The irregular attenuating behaviour is a consequence of the fact that the amplitude of the carrier wave do not steadily go to zero, rather it fluctuates. The fluctuation is due to the constructive and destructive interference of both the 'host wave' and the 'parasitic wave'. In the regions where the amplitude of the carrier wave is greater than either of the amplitude of the individual wave, we have constructive interference, otherwise, it is destructive.

It is clear from fig. 3.1 that the decay process of the maximum displacement or maximum amplitude of the CW is exponential in shape and initially the decay frequency is very rapid. The initial decay frequency indicate the rate at which the initial active quantitative characteristics of the 'host wave' is been destroyed by the interfering 'parasitic wave'. The decay process of the maximum displacement y_m of the CW becomes steady after about 2000s and finally it goes to zero after 8251s.

The steady decay process of the CW signifies that all the active quantitative components of the 'host wave' would have been completely eroded by the interfering 'parasitic wave' and since the 'parasitic wave' does not have its own physical parameters to sustain a continuous independent existence it finally goes to zero after a given period of time. However, in the absence of the 'parasitic wave' (λ = 0) the maximum displacement curve does not attenuate to zero easily under the same condition. The 'host wave' is continuous although it does not fluctuate beyond 2400s and this information is shown by the blue upper curve of fig. 3.1.

From the result of our calculation, it is revealed that the maximum amplitude or the maximum displacement of the carrier wave is made up of both the imaginary and the real part; $A = A_1 + iA_2$. This shows that the motion of the carrier is actually two-dimensional (2D). Thus A_1 and A_2 are the components of the amplitude in x and y - directions and A is tangential to the path of the moving amplitude in the carrier wave. The imaginary value of the maximum displacement of the carrier wave which occurs at $\lambda = 0$, 8, 9, 12, 14, e.t.c., is unnoticeable or inadequately felt by the physical system described by the carrier wave.

Although, unnoticeable as it may, but so much imaginary destructive harm would have been done to the intrinsic constituent parameters of the 'host wave'. It should be noted here that we only used the absolute values of the corresponding maximum displacement. The value of the maximum displacement of the carrier wave for the said imaginary values of the multiplier λ is assumed to be negative instead of the imaginary value ($i = -1$). Then the maximum displacement is plotted against time.

However, beyond this complex anomalous interval, the amplitude of the carrier wave begins to fluctuate with positive values. In this region, the intrinsic parameters of the 'host wave' in the carrier wave are putting a serious resistance to the destructive influence of the 'parasitic wave'. This resistance is an attempt by the constituent parameters of the 'host wave' to annul the destructive effects of the 'parasitic wave', thereby restoring the system of the 'host wave' back to the original activity and performance as it possessed initially. If the restoring tendency of the constituent parameters of the 'host wave' is not effective enough, then the amplitude of the carrier wave depreciates or decays gradually to zero and it ceases to exist.

It is shown in fig. 3.2 that initially the spectrum of the total phase angle of CW has a very high frequency and hence a small wavelength between 0 – 2000s. This is however followed by irregular low frequency and longer wavelength. The total phase angle attenuates to zero after 8251s.

It is observed in fig. 3.3 that initially the decay frequency of the characteristic angular velocity of the CW is very high. This indicates that the rate at which the components of the 'host wave' is been destroyed by the interfering 'parasitic wave'. The characteristic angular velocity has initial maximum positive value of 0.838232 rad/s at about 100s after the interference of the 'parasitic wave' on the 'host wave'. The characteristic angular velocity has a maximum negative value of -3.4522 rad/s at 1600s and it finally attenuates to zero after 2000s. Positive radial velocity means attraction and hence constructive interference between the 'host wave' and the 'parasitic wave', while negative radial velocity means repulsion and hence destructive interference between them.

The trend of event with respect to the total phase angle E and the characteristic angular velocity Z of the CW are similar to that of the maximum amplitude or the maximum displacement as discussed above. Since the wave characteristic of the maximum displacement of the CW due to the raising influence of the multiplier is present in the parameters of both E and Z. However, the values of E and Z is greater than those of y_m.

The fundamental velocity of the CW which is the first term of (2.67) is shown in fig. 3.4. It has a positive maximum value of 4.49 x 10^{-12} rad/s and a negative minimum value of -4.2 x 10^{-12}rad/s. The fundamental velocity of the CW goes to zero after 400s. Fig. 3.4 and 3.5 provides how consistently the constituent parameters of the 'host wave' are correspondingly attenuated to zero by the increasing parameters of the 'parasitic wave'.

We should first emphasize here that fig. 3.6 is a continuation of fig. 3.5 and this graph represents the second terms or the summation terms on the right side of (2.67) containing β. We have only stretched it with the interval of the multiplier λ [0, 22] and λ [22, 43.63] in order to reveal clearly some of the significant features which are unveiled if we used one continuous interval of λ [0, 43.63]. Thus generally, the radial velocity of the CW has a positive maximum value of 1.78888 x 10^{-10} rad/s and a negative minimum value of -1.93956 x 10^{-10} rad/s. It has a low decay velocity frequency which is finally brought to rest after about 1600s.

Figs. 3.7 and 3.8 are the graphs of equation (2.68) and they both represent the radial velocity of the CW in the absence of the 'parasitic wave' ($\lambda = 0$). Thus it is the propagation of only the 'host wave'. While fig. 3.7 represents the fundamental radial velocity of the 'host wave' fig. 3.8 is the radial velocity of the 'host wave' when the influence of the interfering 'parasitic wave' is not considered. The fundamental radial velocity of the 'host wave' has a positive maximum radial velocity of 1.5 x 10^{-7}rad/s and a negative minimum radial velocity of -1.2 x 10^{-6} rad/s. The radial velocity of the of the 'host wave' has a positive maximum value of 2.691048 rad/s and a minimum negative value of -2.69194 rad/s. It is clear that the radial velocities bandwidth of the 'host wave' as shown in figs. 3.7 and 3.8 are larger than those of the CW when the influence of the 'parasitic wave' is considered as shown in figs. 3.5 and 3.6.

Initially the decay spectrum of the attenuation frequency of the propagation of the CW is very high but it decreases with increasing wavelength after 2000s. The propagation of the 'host wave' in the absence of the 'parasitic wave' is almost brought to zero after 8251s, that is when the β = 43.63.

It is also shown in fig. 3.9 that the respective distances covered by the CW consistently increases in the situation when the influence of the 'parasitic wave' is not considered but the CW is drastically brought to rest in the case when the multiplier is considered. That means the effect of the 'parasitic wave' has caused a serious retardation on the transport mechanism of the carrier wave after a sufficiently long time and maximum value of β.

We should emphasize here that the take off time cannot be exactly zero; it could be any value different from zero. This is the correction in time. Hence the model produced a value of x = 0.02m for the linear distance covered even at t = 0 and λ = 0. That is, at t = 0, 0.51s, 2.05s, 4.66s, 8.39s,…, 3964.97s, 8251.37s and with a corresponding value of λ = 0, 1, 2, 3, 4,…, 44, 43.63, the linear distance covered by the CW given by (2.75) are x ; 0.02m, 1.45m, 5.89m, 13.36m, 24.02m,…,11235.50m, 64.26m respectively. However, in the absence of the multiplier λ which is represented by (2.76), under the same given time, the corresponding linear distance covered by the CW are x ; 0.02m, 1.75m, 5.51m, 12.92m, 23.63m,…,11357.50m, 23635.23m respectively.

Thus when we fixed the Fourier index β at 43.63, then the total relative distance covered by the CW in the absence of the multiplier is 109896m and with a total time of 38367s while in the presence of the multiplier the CW covered a total relative distance of 86047m with the same time. The difference in the total relative distance covered is 23849m, that means the expected total relative distance to have been covered by the 'host wave' is now altered or reduced by the influence of the 'parasitic wave' by 22%.

5.0 Conclusion

All physical systems are guided by some inbuilt internal factors that annul the destructive influence of any external interfering wave. The interference of a 'parasitic wave' on a 'host wave' could cause the 'host wave' to decay to zero if they are out of phase. The decay process of the 'host wave' can be gradual, over-damped or critically damped depending on the rate in which the amplitude of the 'host wave' is brought to zero. However, the general concept is that the intrinsic parameters of the 'host wave' in the carrier wave would first put a serious resistance to the destructive influence of the 'parasitic wave'. This resistance is an attempt by the constituent parameters of the 'host wave' to annul the destructive effects of the 'parasitic wave', thereby restoring the system of the 'host wave' back to the original activity and performance. If the restoring tendency of the constituent parameters of the 'host wave' is not effective enough, then the amplitude of the carrier wave depreciates and decays gradually to zero.

5.1 Suggestions for Further Work

This study in theory and practice can be extended to investigate wave interference and propagation in three-dimensional (3D) system. The carrier wave we developed in this work can be utilized in the deductive and predictive study of wave attenuation in exploration geophysics and telecommunication engineering.

Appendix

The following is the list of some useful identities which we implemented in the study.

1. $\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}$; (2)

$\sin x - \sin y = 2\cos\frac{x+y}{2}\sin\frac{x-y}{2}$

(3) $\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$; (4)

$\cos x - \cos y = -2\sin\frac{x+y}{2}\sin\frac{x-y}{2}$

(5) $2\sin x\cos y = \sin(x+y) + \sin(x-y)$; (6)

$2\cos x\sin y = \sin(x+y) - \sin(x-y)$

(7) $2\cos x\cos y = \cos(x+y) + \cos(x-y)$; (8)

$2\sin x\sin y = \cos(x-y) - \cos(x+y)$

(9) $\sin(x \pm y) = \sin x\ \cos y \pm \cos x\ \sin y$; (10)

$\cos(x \pm y) = \cos x\ \cos y \mp \sin x\ \sin y$

(11) $\sin 2x = 2\sin x\cos x$;

(12) $\sin(-x) = -\sin x$ (odd and antisymmetric function)

(13) $\cos(-x) = \cos x$ (even and symmetric function)

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